Name: Feedback

SA402 · Dynamic and Stochastic Models Fall 2022 · Uhan

Quiz 8 – 11/30/2022

Instructions. You have 20 minutes to complete this quiz. You may use your plebe-issue calculator and the provided list of formulas for birth-death processes. You may not use any other materials (e.g., notes, homework, books).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1a	1	
1b	1	
3	2	
4	1	
Total		/ 50

Problem 1. Four Guys Burgers and Fries has 3 cashiers at its Simplexville location. Customers wait in a single queue and are served by the first available cashier, first-come first-served. The cashier area is relatively small, and can only hold 10 customers (including the 3 receiving their orders). Any customers that arrive when the cashier area is full simply go elsewhere. The average service time is 5 minutes per customer, and customers arrive at a rate of 20 per hour. Assume the interarrival times and the service times are exponentially distributed.

Model this setting as a birth-death process by answering the following prompts.

a. Define the arrival rate in each state, in terms of the number of customers per hour.

For a similar example, see Case 3 in Section 4 of Lesson 12.

Note that the system can hold 10 customers. So, when there are 10 customers in the system, what should the arrival rate into the system be?

b. Define the service rate in each state, in terms of the number of customers per hour.

For a similar example, see Case 4 in Section 5 of Lesson 12.

Problem 2. Customers call the reservation desk at Fluttering Duck Airlines at a rate of 4 customers per hour. There is 1 agent working at the reservation desk at any given time, and each phone call takes an average of 20 minutes. The phone system can only handle 3 customers at a time (1 with an agent, 2 waiting) – any phone calls arriving when there are 3 customers are simply lost.

This setting can be modeled as a birth-death process with the following arrival and service rates:

$$\lambda_i = \begin{cases} 4 & \text{if } i = 0, 1, 2 \\ 0 & \text{if } i = 3, 4, \dots \end{cases} \qquad \mu_i = 3 \quad \text{for } i = 1, 2, \dots$$

a. Over the long run, what is the probability that there are n customers in the system? ($n = 0, 1, 2, 3$)

Most of you had the right idea here. Be careful when using the formulas for the steady-state probabilities. See Example 1 of Lesson 13 for a similar problem.

b. Suppose you find that the steady-state probabilities are:

$$\pi_0 = 0.15 \qquad \pi_1 = 0.21 \qquad \pi_2 = 0.27 \qquad \pi_3 = 0.37 \qquad \pi_n = 0 \quad \text{for } n = 4, 5, \dots$$

(This may or may not match what you found in part a.) Over the long run, what is the expected number of customers in the queue?

See Example 2 of Lesson 13 for a similar problem. Also, note that you are given values of π_j for this part of the problem – you do not need to rely on your answers to part a.